

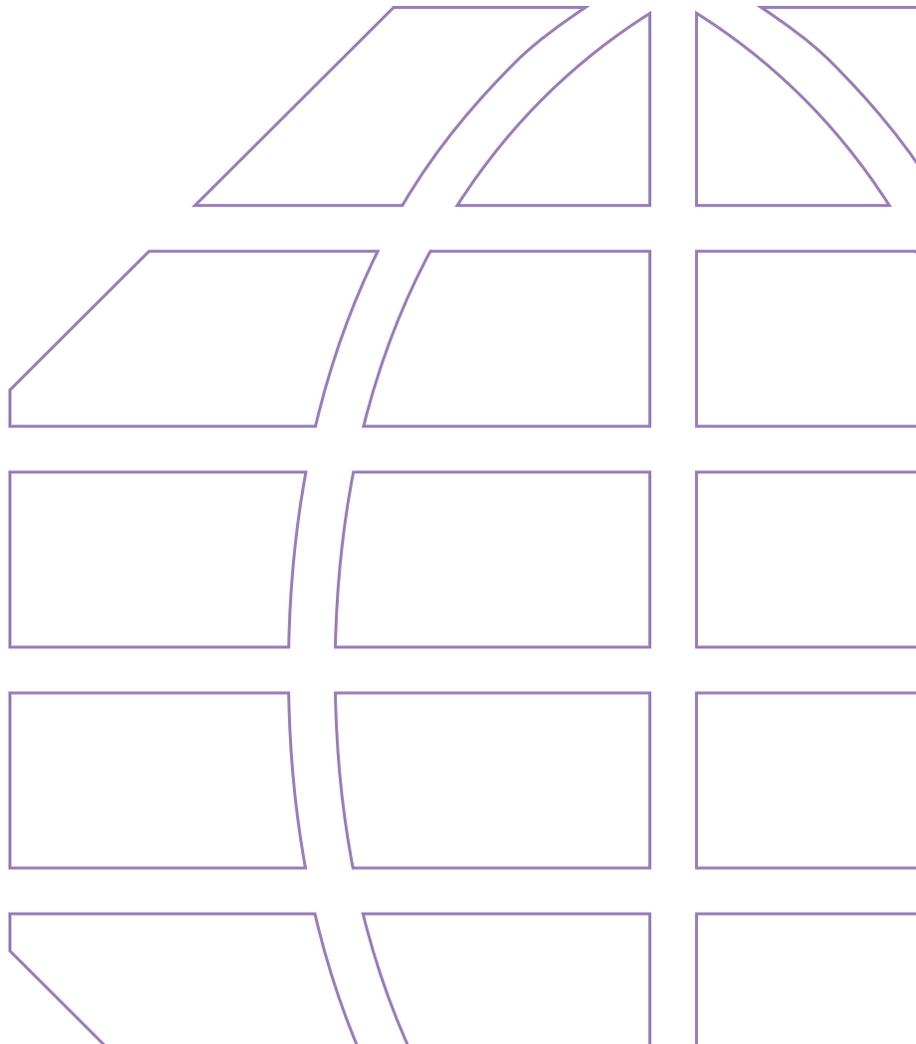
RESEARCH AND PRODUCT DEVELOPMENT

# Leveraged ETFs vs. Futures: Where Is the Missing Performance?

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## This article examines under-performance and daily rebalancing issues associated with leveraged ETFs, and possible solutions using equity index futures and options.

Leveraged ETFs have garnered plenty of attention of late, for a couple of reasons: They provide an alternative way for investors to deploy leverage in their investment portfolio. They also provide investors a way to “short” the market, either as a hedging or speculative tool. One of the knocks on leveraged ETFs is the observation that they tend to underperform their respective indexes (after adjusting for their leverage level). In this article, we highlight one very important source of the under-performance, and discuss some possible remedies using equity index futures and options.

### Daily Rebalancing of Leveraged ETFs

One of the most important features of a leveraged ETF is the necessity to “rebalance” on a daily basis. To illustrate: Assume that we achieved 200 percent of the daily performance of the S&P 500 Index. (We will use the S&P 500 Index as the underlying index for our discussion, although the same argument would apply to any other index or portfolio, or even to commodities.)

At the conclusion of any given day, the leverage in the portfolio most likely will be something other than the “2 times” prescribed for the portfolio. If the index was up on the day, the “equity” in the portfolio would have increased dollar for dollar with the “2-beta” portfolio. The portfolio would be under-leveraged for the following day. As such, the manager would need to buy more exposure in the underlying index, by buying index futures, or buying more stocks for the portfolio, etc.

Likewise, following a down day, the portfolio would be over-leveraged, forcing the manager to reduce the exposure to the index by selling stocks or futures, etc.

This daily rebalancing act enables the portfolio to deliver the leveraged performance on a daily basis. It also, however, creates a rather interesting phenomenon – buying after the index has gone up, selling after the index has fallen.

The effect of this phenomenon would be most noticeable when the market is trading “sideways.” Consider the following extreme case, in which the market vacillates between two levels in consecutive days. The manager would buy at the end of the “up day,” only to turn around and sell at the end of the next day at a lower price. The result: The portfolio would be bleeding slowly over time. Meanwhile, the market is stuck in place at the same general level.

The same dynamic would be at work for the “short” ETFs as well. If the index drops, the equity in the overall portfolio would increase disproportionately, necessitating more “shorting.” These positions would be bought back after the market goes up. To summarize, the daily rebalancing act would force the manager to chase his/her position, regardless of the direction of the leverage.

### Simple Mathematical Argument

A little rudimentary arithmetic offers a slightly more in-depth perspective. Assume that we are holding the leveraged ETF for  $N$  days. Further, we use  $\beta$  to denote the desired leverage. For our 200 percent daily exposure,  $\beta = 2$ . We denote the underlying daily index performance as  $r_i$  for the  $i^{\text{th}}$  day. The value of the portfolio after the  $N$  days would be given by:

$$V = \prod_{i=1}^N (1 + \beta r_i)$$

This is derived by nothing more than compounding the daily returns of the index on  $N$  consecutive days. This equation can be recast in logarithmic terms as follows:

$$V = \exp \left[ \sum_{i=1}^N \ln (1 + \beta r_i) \right]$$

If we apply a second-order Taylor series expansion and approximate the right-hand side of the equation, we will discover that the value is approximately given by:<sup>1</sup>

$$V \approx (1 + \beta \bar{r})^N \exp \left[ -\frac{1}{2} \frac{\beta^2}{(1 + \beta \bar{r})^2} N \sigma^2 \right]$$

where  $\sigma^2 = \frac{1}{N} \sum_{i=1}^N (r_i - \bar{r})^2$ . This expression is less daunting than it appears.  $\bar{r}$  denotes the average daily return for the index during the period. The first half of the expression is no more than the compounded returns based on the average daily performance over the period. This compounded daily return is scaled by the second term that has a magnitude smaller than 1. This is where the under-performance comes in. Notice that  $\sigma^2$  is the sample variance of the daily return. The “correction” has to do with the volatility of the market during the holding period.

From this approximation, we can easily verify that in a sideways (i.e.,  $\bar{r} \approx 0$ ) but very volatile stretch of holding period in the market, the value of the portfolio would be lower than the initial value, represented as ( $\approx \exp(-\frac{1}{2}N\beta^2\sigma^2)$ ), and would actually decline significantly. The severity of this decline would depend on the magnitude of the realized variance of the underlying index, as well as the level of leverage deployed.

### Evidence of the Gamma Losses

The phenomenon described above is very reminiscent of the balancing act that option traders face every day – a short delta-neutral option position will carry a “negative gamma,” meaning that the loss is proportional to the square of the changes in the underlying price – offsetting the time decay of the option value.

For brevity, we will call this variance-related loss the **gamma loss**. To verify the occurrence of this gamma loss for leveraged ETFs, we can conduct the following

thought experiment. At the end of each month, we initiate positions in the 2-beta as well as the minus-2-beta S&P 500 ETFs in equal dollar amounts, using the ProShares Ultra and UltraShort ETF family. We tally the value of the two positions at the end of the month, including all the distributions during the month (see Exhibit 1 below).

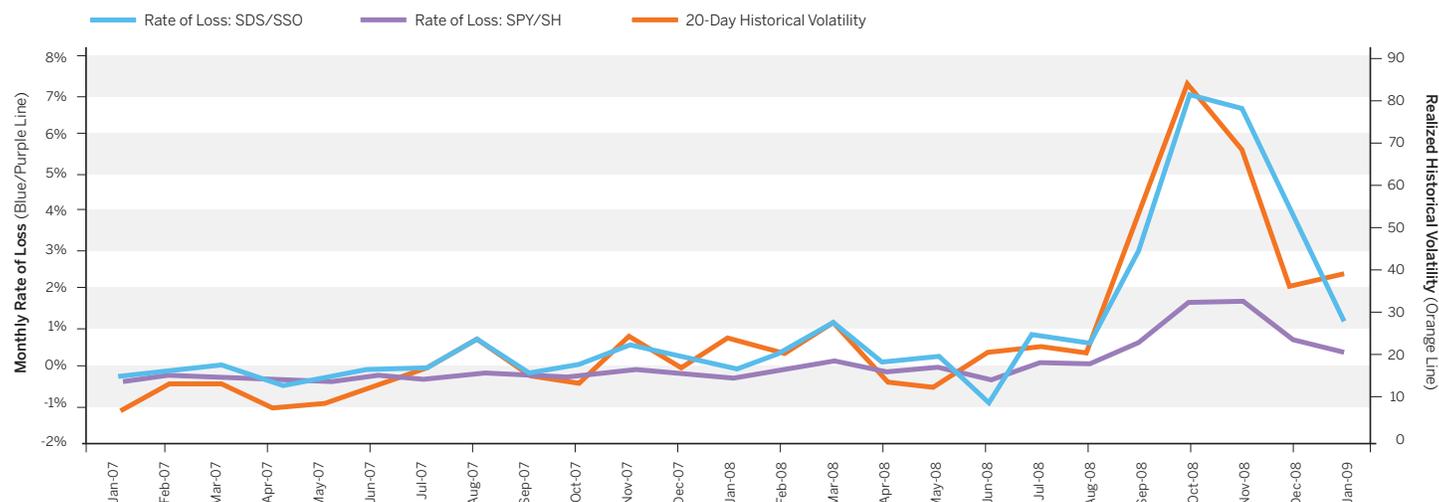
Clearly, the portfolio is supposed to be self-hedging. With equally weighted plus and minus 2-beta halves in the portfolio, we would have expected the monthly performances of the opposing ETFs to approximately offset. The graph in Exhibit 1 shows that it is decidedly not so. In volatile months like those in the third and fourth quarter of 2008, the self-hedged portfolio of the leveraged ETFs actually delivered net losses upwards of 7 percent in a month!

Furthermore, the rate of loss for the ETF pair correlates very well with the realized volatility of the S&P 500 Index.

To verify that the gamma loss is proportional to the square of the beta in the leveraged portfolio, we also assembled the performance of the portfolio with 1-beta and minus-1-beta. The purple line in Exhibit 1 shows that the gamma loss follows the trends in the leveraged ETF pair, albeit at a much lower magnitude. In fact, during the peak months for the gamma losses, the leveraged ETF pairs spotted 4 times as much in gamma losses as its one-beta counterpart.<sup>2</sup> In other words, the gamma losses scaled the way our derivation had suggested.

### EXHIBIT 1:

Monthly Rate of Loss in the Leveraged ETF pairs vs. realized 20-day Historical Volatility of the S&P 500 Index. SDS and SSO represent the ProShares UltraShort S&P 500 and Ultra S&P 500 ETFs, respectively. SPY and SH represent the SPDR S&P 500 and ProShares Short S&P 500 ETFs, respectively.



In volatile markets (similar to those in the third and fourth quarter of 2008), the self-hedged portfolio of leveraged ETFs actually delivered net losses upwards of 7 percent in a month.

#### Implications for Choosing Leveraged Instruments

Some noteworthy observations can be made:

- The gamma losses we have discussed in this article are attributed to the fact that daily rebalancing is necessary to the survival of leveraged ETFs. In the very short run, investors would not notice the gamma losses, for the losses will only accumulate over time and tend to be masked by the directional P/L – investors typically do not hold such self-hedged ETF portfolios.

- If the holding period for the position is longer, the choice of instrument for the purpose of employing leverage becomes important. There is a perfectly simple way of avoiding such gamma losses – using index futures. Except for the cases of margin calls, perhaps, portfolio managers have the discretion to manage the hedge as well as the frequency of rebalancing. In volatile markets, controlling such gamma losses can prove to be very valuable.
- Further, market participants can manage the gamma losses with other listed products, e.g., option straddles and strangles. In the cases in which there is no futures market in the underlying index, buying volatility via listed options in correlated indexes might be the next best thing on the menu. For example, there are no liquid index futures for the financial sector. However, one can still buy back some of the volatility via S&P 500 option strategies.

## For More Information

If you have questions or want more information on how to apply CME Group equity index futures and options to manage gamma losses, contact the Equity products team today:

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1. The approximation here is, by design, very crude in nature. There are various ways to improve upon the exactitude of the expression. Since we have already accomplished what we intend to demonstrate, we forego the additional steps to tidy up the loose ends.
2. The one-beta ETF is, of course, not leveraged. The self-hedged pairing of it to the short ETF is constructed to demonstrate the relative magnitude of the gamma losses due to the choice of beta. Therefore, it is obvious that the short ETF bears the blunt of the gamma loss.

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